Communication Index

a study of a new efficiency measure for networks – *work in progress !!!*

involved so far:

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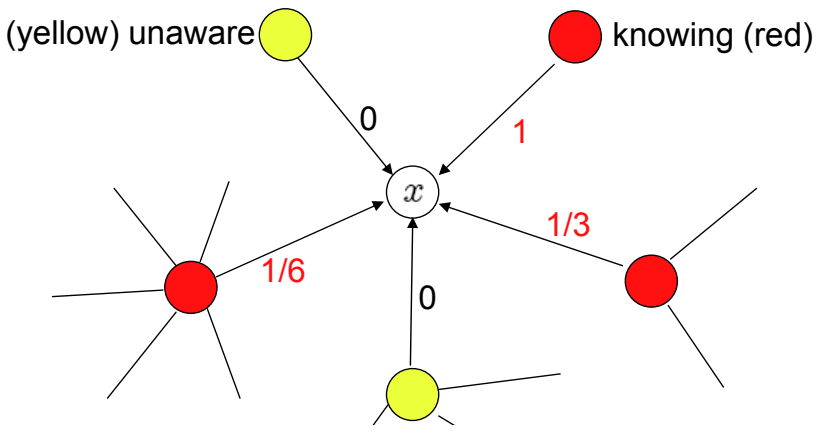
Where does it come from …?

GEP modell for knowledge diffusion on networks …
… the local interaction depends on how busy s.o. is

local observables

$\Omega_t(x) = \sum_{x \sim y} \omega(y)$ number of knowing neighbours of x **= 3**

$\Phi_t(x) = \sum_{x \sim y} \frac{1}{d(y)} \omega(y)$ local knowledge inflow **=1+1/3+1/6 =1.5**



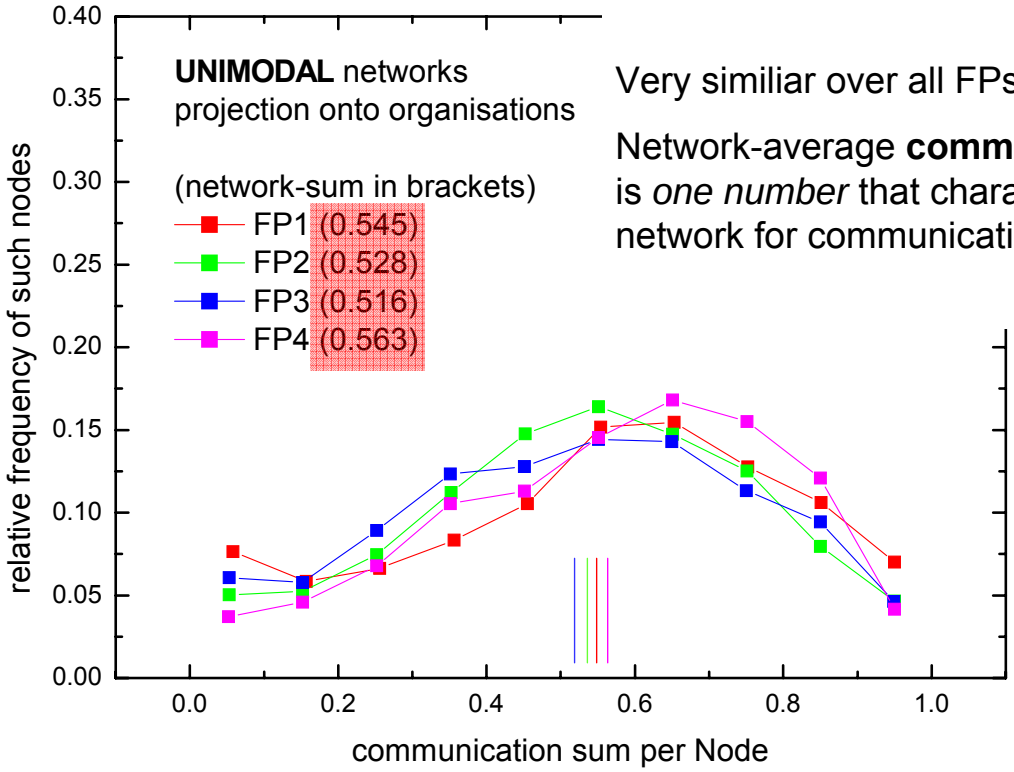
1/degree weighing of the knowing neighbours

From that *process* to a *static* measure

- Let an existing edge x~y symbolize *communication* between node x and y
- Time someone *can* spend with neighbours be equally divided among them → 1/degree
- BUT: Relevant for the time that is *actually spent* …
- … is the *more busy* of both nodes:
  → edgeweight(x,y) = min [1/degree(x), 1/degree(y)]
- Sum of all such edgeweights around each x: communication „capacity utilisation“ („workload“) of x
- then Sum this over all nodes / take the average =: „communication index“ of whole network

UNIMODAL ORGs Projection (unweighted)

Node statistics of „capacity utilisation“ („workload“)

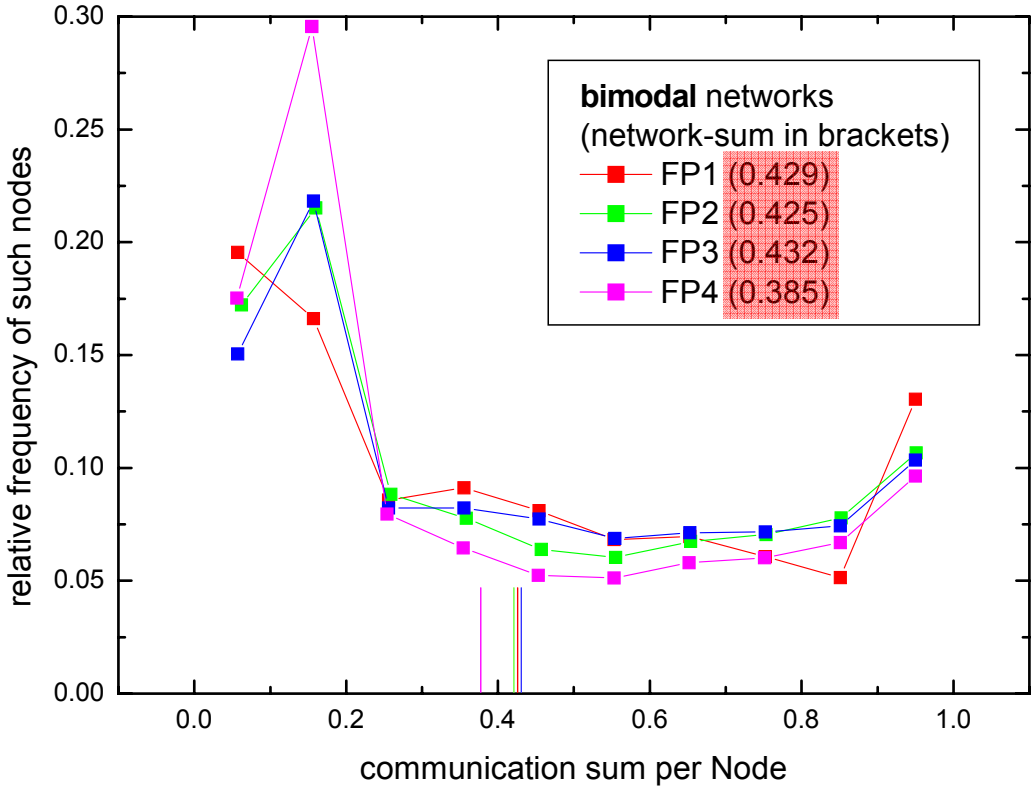


Very similiar over all FPs !!!

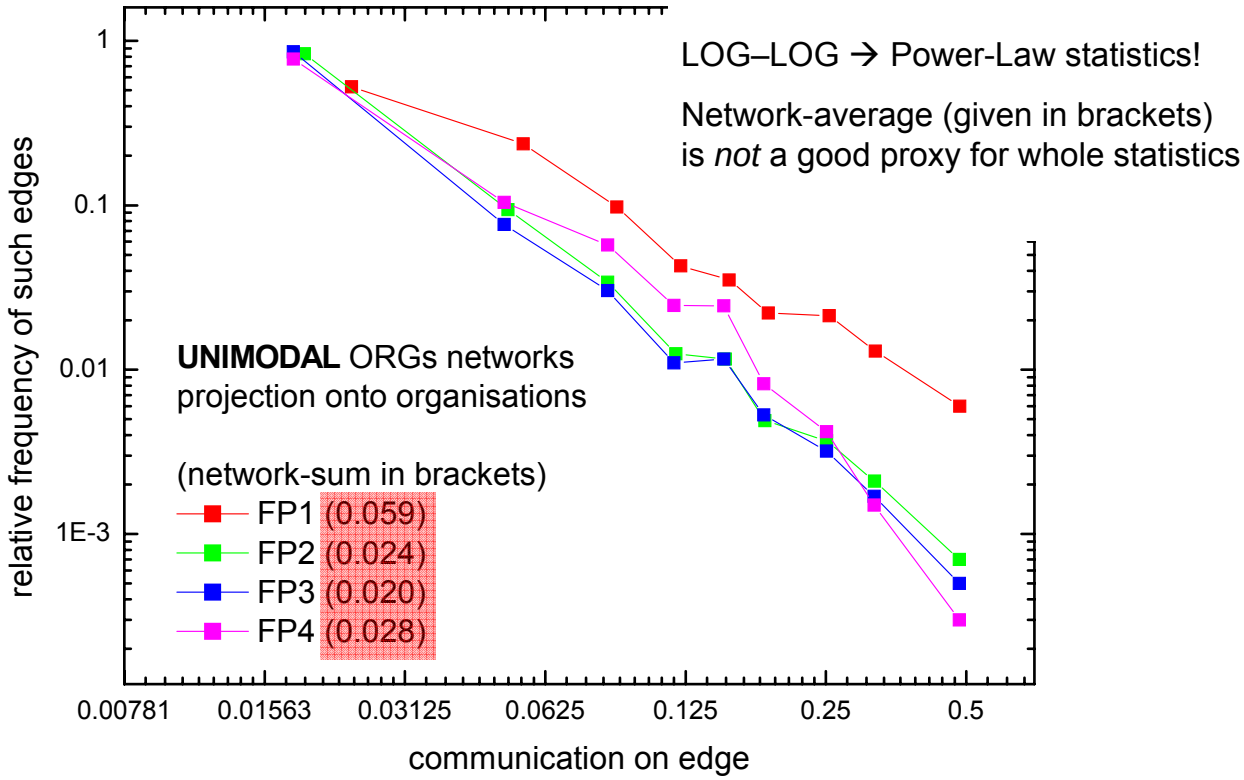
Network-average **communication-index** is *one number* that characterizes the network for communication purposes

BIMODAL (projects also treated as actors!)

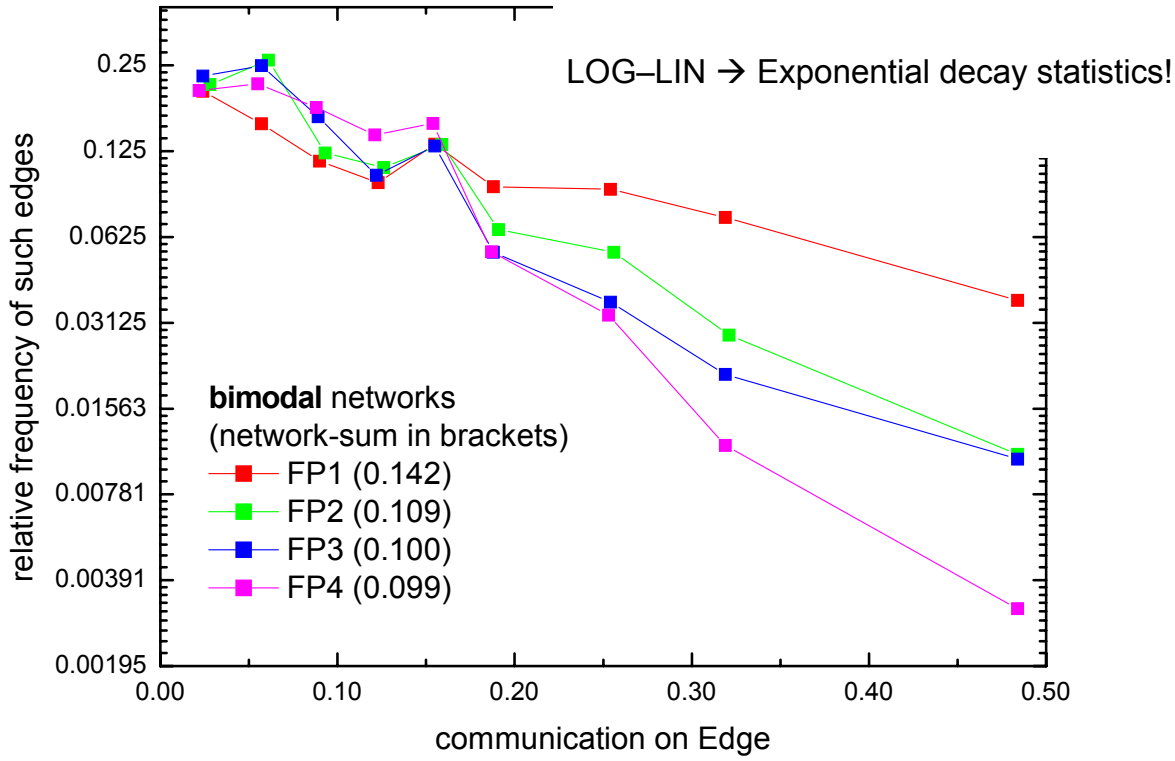
Node statistics of „capacity utilisation“ („workload“)



UNIMODAL ORGs Projection (unweighted) communication-edgeweights statistics



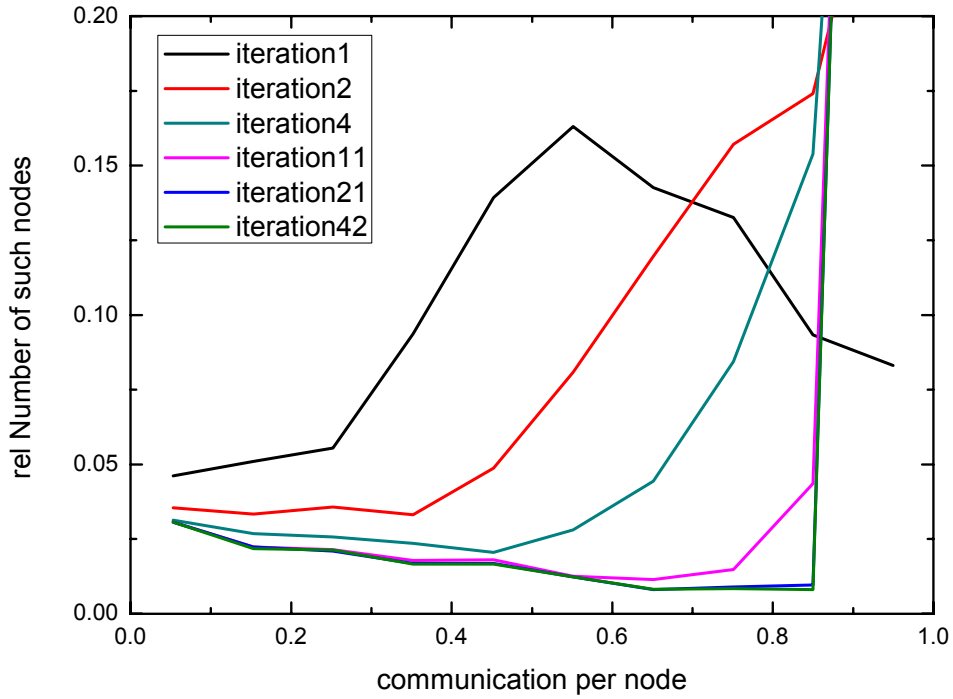
BIMODAL (projects also treated as actors!) communication-edges statistics



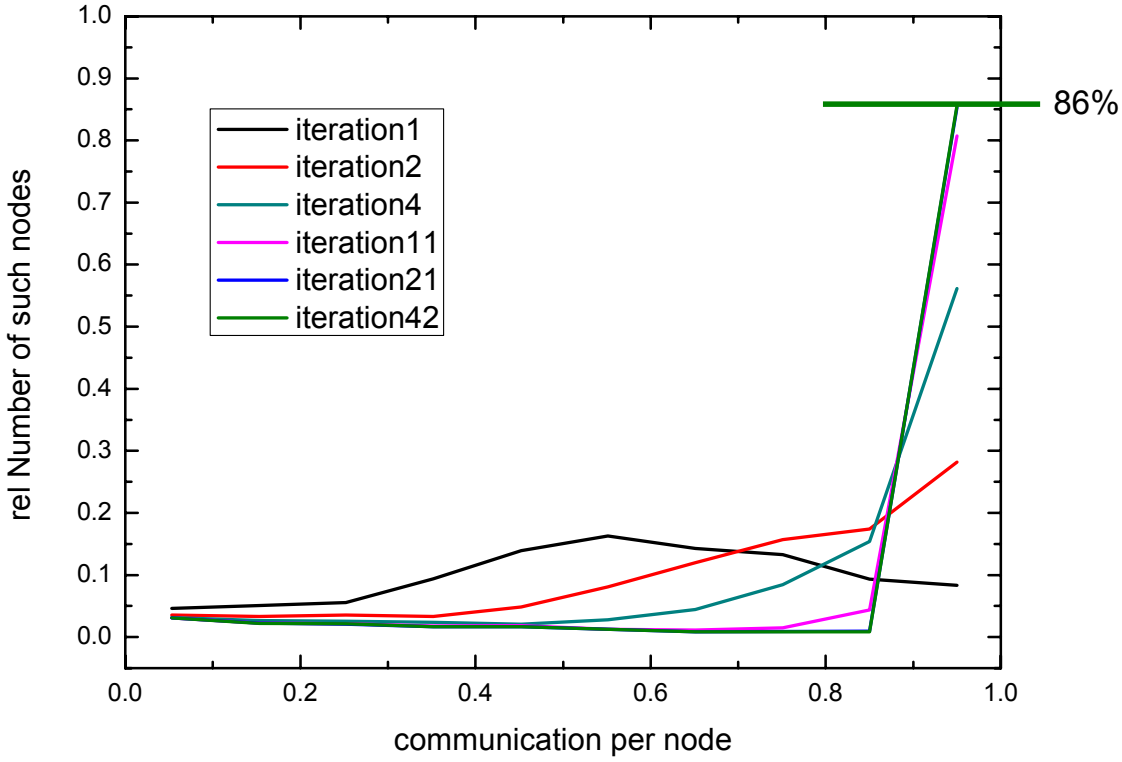
Further iterations

- The unbusy nodes still have free communication capacity among each other
- The busy nodes (nodeSum=1.0) are taken out of the game … then it is iterated
- At some iteration, it stagnates.
- Interesting question: How many of the nodes have ~100% communication after stagnation

Iterations until stagnation (FP2_ORGS)



Iterations until stagnation (FP2_ORGS)



Analytically tractable model !

→ Bollobas-Riordan Kernel Method
→ Sascha, Tyll, Madeleine, Philippe
→ Andreas: Mathematica numerics, EVs and plots

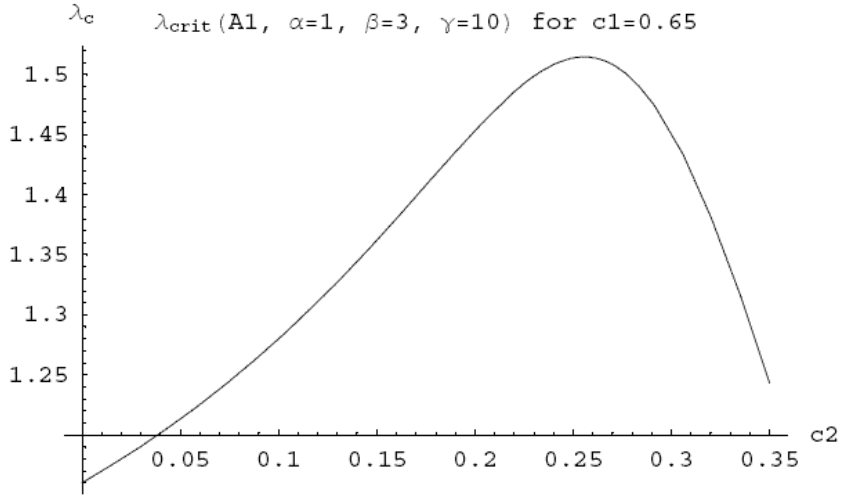
e.g. 3 node types society with mixture of hubs, middle-degree, low-degree :

1) Setup the kernel for 1/degree communication with a knowledge transmission probability λ
2) If Operator-norm of that kernel reaches 1 → birth of giant component
3) For which λ_{crit} does it happen?

Resulting plot, *very* preliminary:

critical transmission probability λ_{crit}

ratios of the *degrees* of the 3 node types = **α:β:γ**

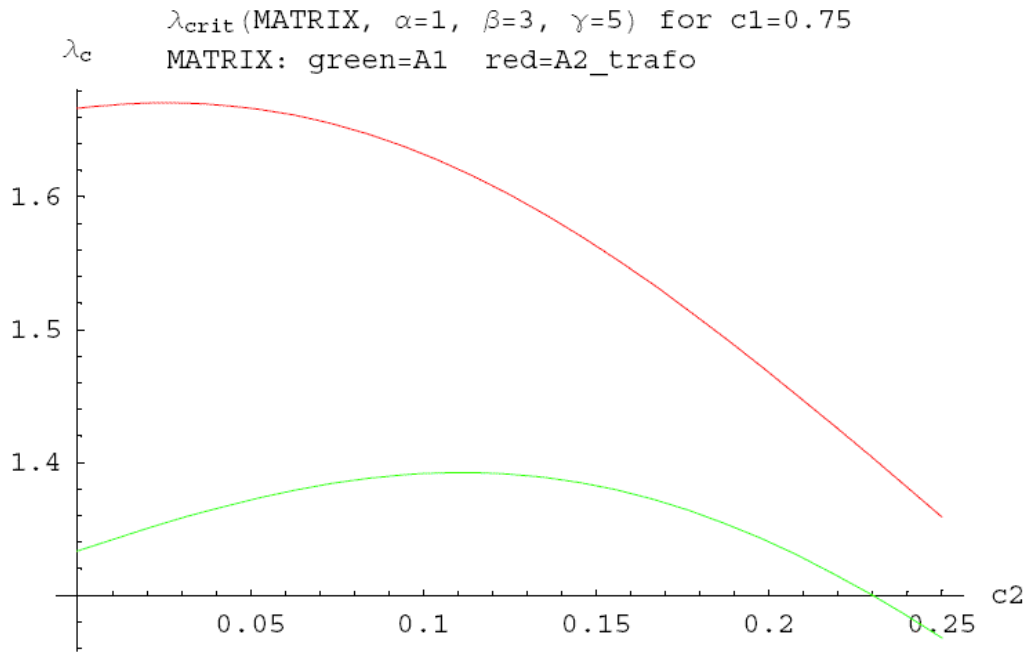


c1 of α-degree-type c2 of β-degree-type c3 of γ-degree-type

c1 fixed to 65% of nodes plot over c2 → c3 = 1 - c1 - c2

So to the right: more hubs, to the left: more middle-degree nodes

Resulting plot, *very* preliminary
Multiplicative vs additive coupling
(green) (red)



Resulting plot, *very* preliminary
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