Komplexe Netzwerke Seminar
`ss2008_289971`

Introduction to Graphs, and the Physics „hype“ about Complex Networks

part 1 of 2

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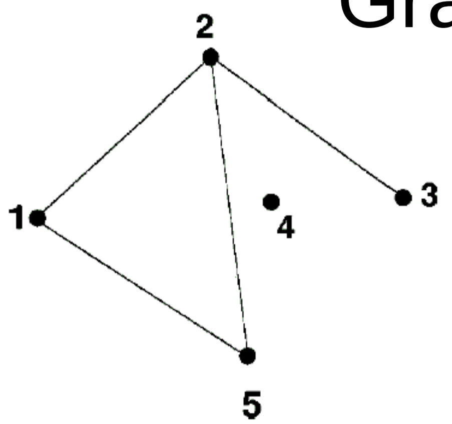
Graph G=(V,E)



- KNOTEN, node, vertex, actor, point, individual ...

- KANTE, edge, line, bond, tie, connection...

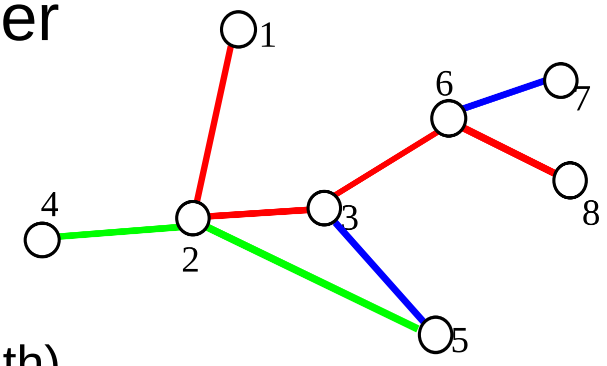
V $\subseteq Z^+$ elements represented by dots
e.g. **V**={1,2,3,4,5}

E $\subseteq \binom{V}{2}$, elements represented by lines
e.g. **E**={ (1,2) , (1,5) , (2,3) , (2,5) }

Neighbours:

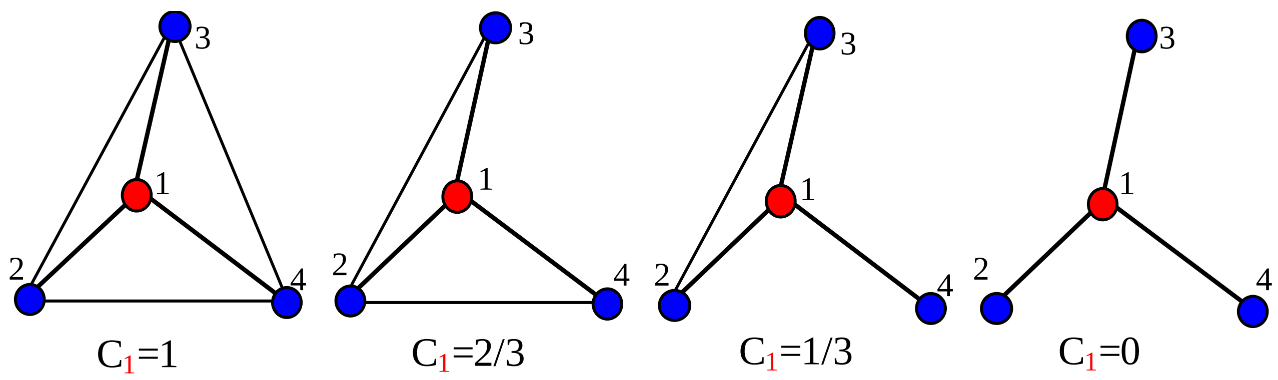
x~y := {$\exists$ **e**$\in$**E** with e=(x,y) or e=(y,x)}

Pfade, Durchmesser



- pathlength (geodesic path)
  - **Shortest** connection between 2 nodes
  - Example pathlength(1,8) = 4
- global graph-properties
  - *Diameter* = **longest** geodesic path (here 4)
  - *characteristic pathlength* = **average** of all paths (i,j)

Cluster-Coefficient, Dreiecks-Zahl



$$C_i = \frac{\#T_i}{k_i(k_i - 1)/2}$$

$$C = \frac{1}{N}\sum_i C_i$$

(global)

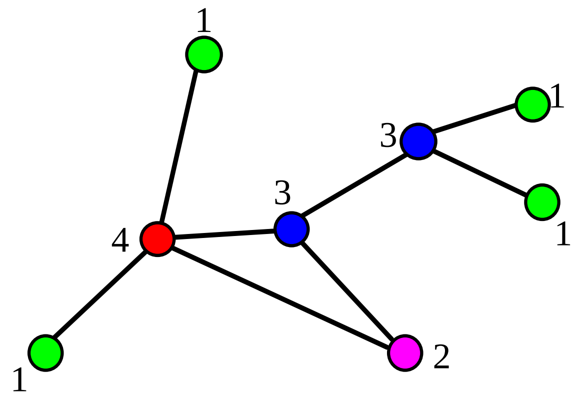
$\#T_i$ = Number of Triangles around vertex i

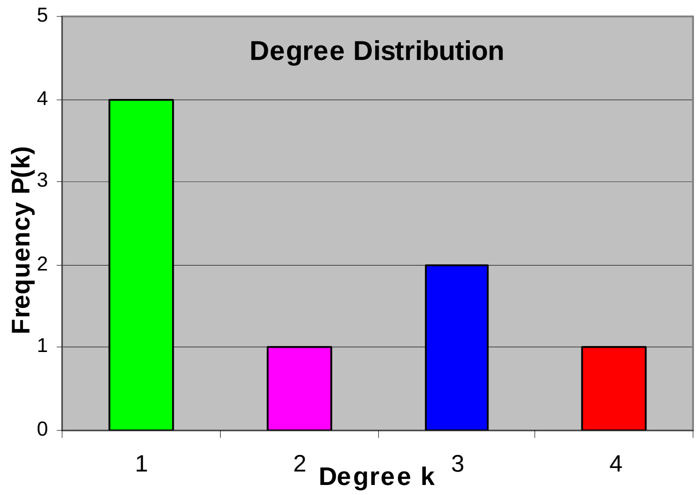
C_i: Estimator for **local density of connections**, “*how many of my friends are friends to each other?*”

degree of a node

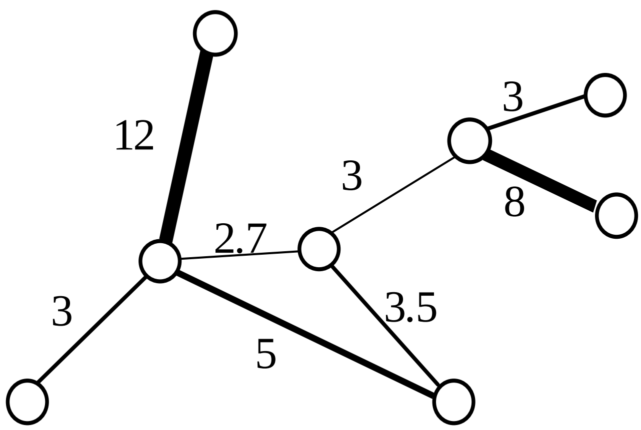
$k_x = deg(x) = |N_1(x)|$
= number of N_1-Neighbours of node x

P(k) = *Degree-Distribution (frequency)*
= **number of nodes** with deg=k



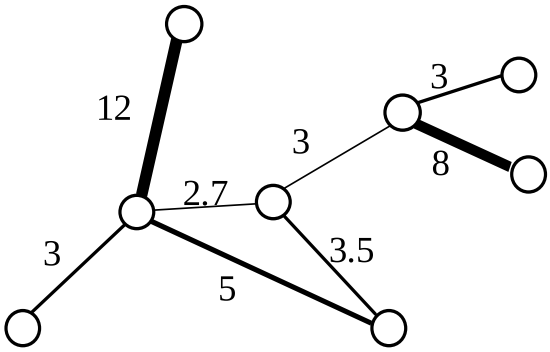


Gewichtete Kanten, gewichteter Graph



Funktion g(e) mit g: E →|R
Gibt jeder Kante eine Stärke, Intensität, etc.
(*ungewichtet*: g(e)=1 für alle e)

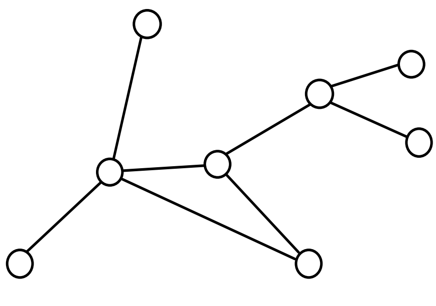
Affinity / Adjacency Matrix

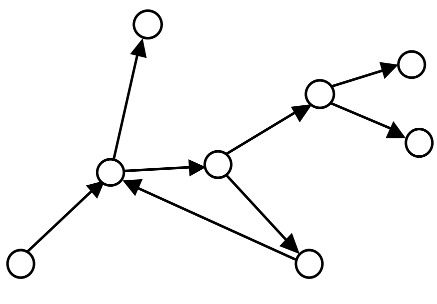


	1	2	3	4	5	6	7	8
1		**3**	0	0	0	0	0	0
2	**3**		**12**	**2.7**	**5**	0	0	0
3	0	**12**		0	0	0	0	0
4	0	**2.7**	0		**3.5**	**3**	0	0
5	0	**5**	0	**3.5**		0	0	0
6	0	0	0	**3**	0		**3**	**8**
7	0	0	0	0	0	**3**		0
8	0	0	0	0	0	**8**	0	

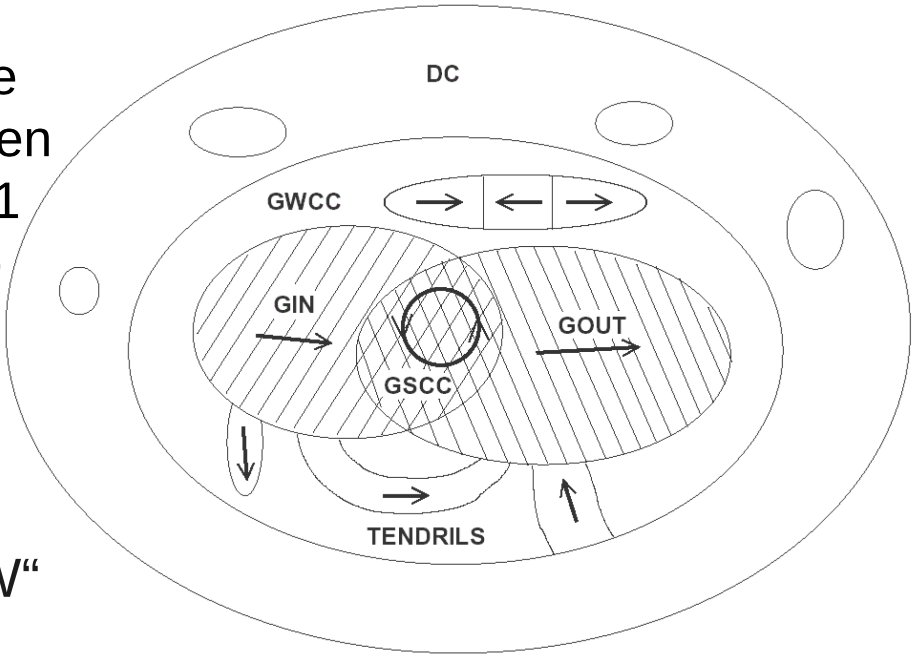
(un)gerichteter Graph, (un)gerichtete Kanten

$$\mathbf{x} \leftrightarrow \mathbf{y} \quad = \quad \mathbf{x} \rightarrow \mathbf{y} \quad \wedge \quad \mathbf{y} \rightarrow \mathbf{x}$$





Struktur im Falle ***gerichteter*** Kanten (**von** Webseite 1 **zu** Webseite 2)



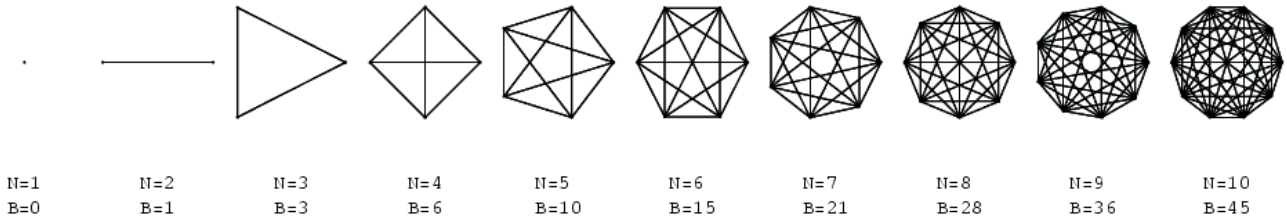
“Bild vom WWW“

FIG. 6. Structure of a directed graph when the giant strongly connected component is present [112] (see the text). Also, the structure of the WWW (compare with Fig. 9 of Ref. [6]). If one ignores the directedness of edges, the network consists of the *giant weakly connected component* (GWCC) — actually, the usual percolating cluster — and disconnected components (DC). Accounting for the directedness of edges, the GWCC contains the following components: (a) the *giant strongly connected component* (GSCC), that is, the set of vertices reachable from its every vertex by a directed path; (b) the *giant out-component* (GOUT), the set of vertices approachable from the GSCC by a directed path (includes the GSCC); (c) the *giant in-component* (GIN), contains all vertices from which the GSCC is approachable (includes the GSCC); (d) the *tendrils* (TE), the rest of the GSCC, i.e. the vertices which have no access to the GSCC and are not reachable from it. In particular, this part includes something like “tendrils” [6] but also there are “tubes” and numerous clusters which are only “weakly” connected. Note that our definitions of the GIN and GOUT differ from the definitions of Refs. [6,61]: the GSCC is included into both GIN and GOUT, so the GSCC is the interception of the GIN and GOUT. We shall show in Sec. XIB that this definition is natural.

Clique = FullGraph

Anzahl möglicher Kanten zwischen N Knoten:

$$M = \frac{N(N-1)}{2}$$



Bipartite Graphs

Up to now we have only seen so called *1-mode graphs*, i.e. there is **one** type of vertices

Now imagine for example 4 **films** (black) and 11 playing **Actors** (white).

From the 2-mode graph we can generate a 1-mode graph by projection (under information loss)

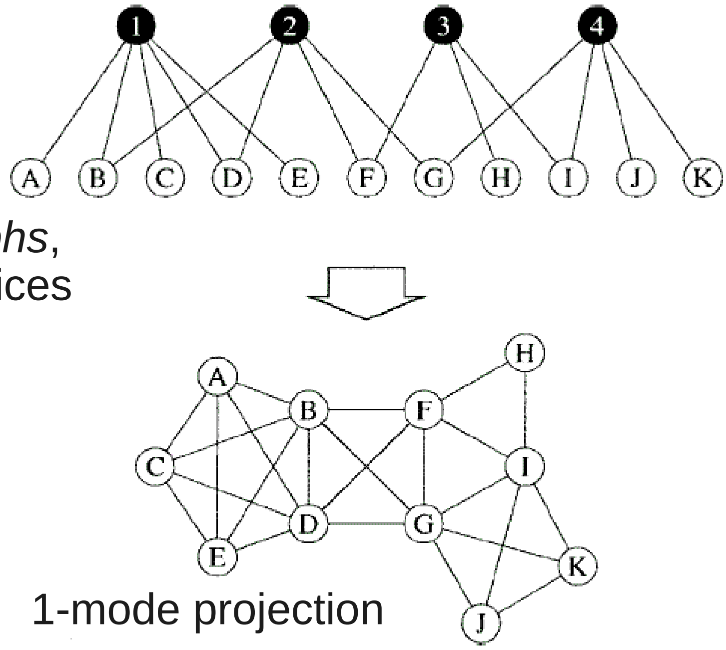
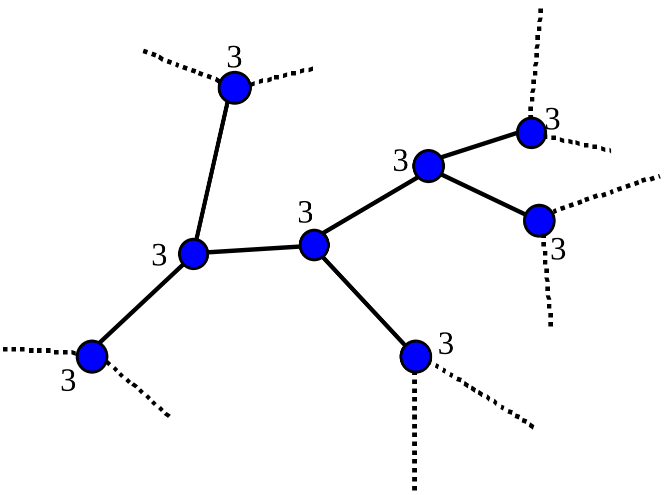
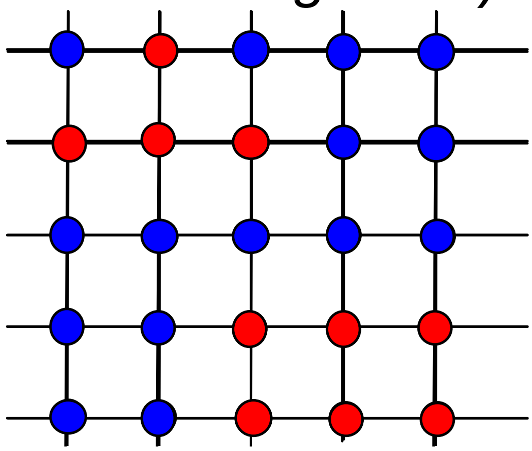


FIG. 14. A schematic representation of a bipartite graph, such as the graph of movies and the actors who have appeared in them. In this small graph we have four movies, labeled 1 to 4, and eleven actors, labeled A to K, with edges joining each movie to the actors in its cast. The bottom figure shows the one-mode projection of the graph for the eleven actors. After Newman, Strogatz, and Watts (2001).

Special cases of graphs: Some Trees, Lattices are *regular* graphs (*around for a long time*!)



Caley-Tree with coordination number (degree) z=3
... *branching process* ...



Lattice Z^2 - some *processes*:

- condensed matter - crystals
- self organized criticality (SOC)
- Ising (model magnet with *spins*)
- ...

Degree ist überall k = 4
Durchmesser ist $\sqrt{2N}$

Teil 2 = nächstes Mal