



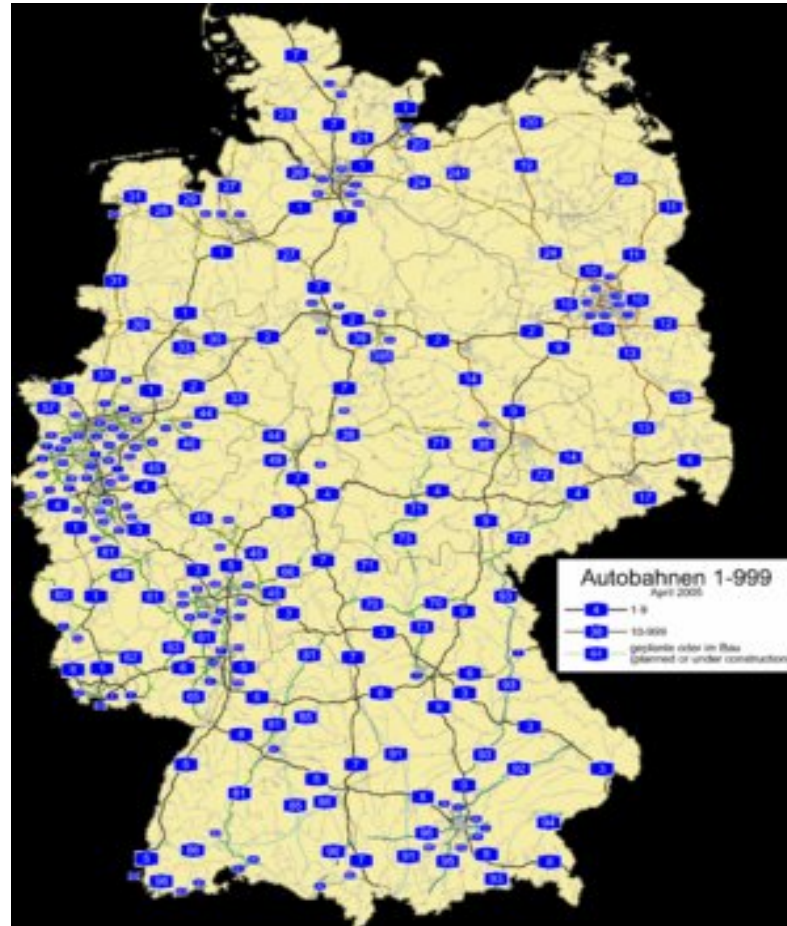
Analyzing the network of German Autobahns

Contents

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- Graph of the network of German Autobahns
- Interests of research
- Spectral properties of the transition operator
- Space syntax representations
- Conclusions



Network of German Autobahns





Typical section of a modern Autobahn near an interchange, with overhead direction signs.



The number signet for the A 8 as it appears on all traffic signs.



Pattern of German Autobahns with numbers 10 to 999

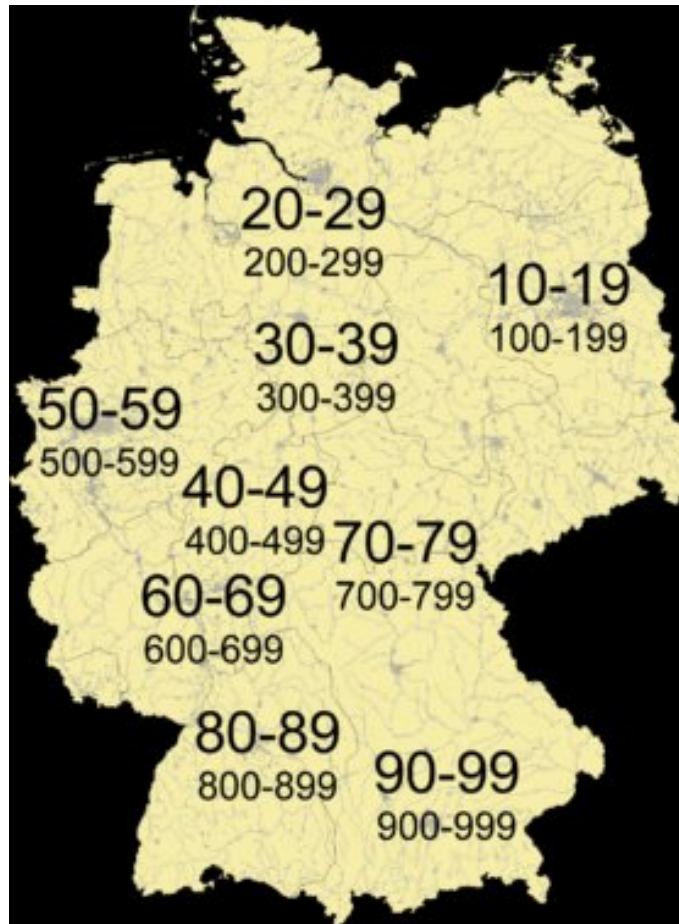




Table of existing German Autobahns

Existing German Autobahns:

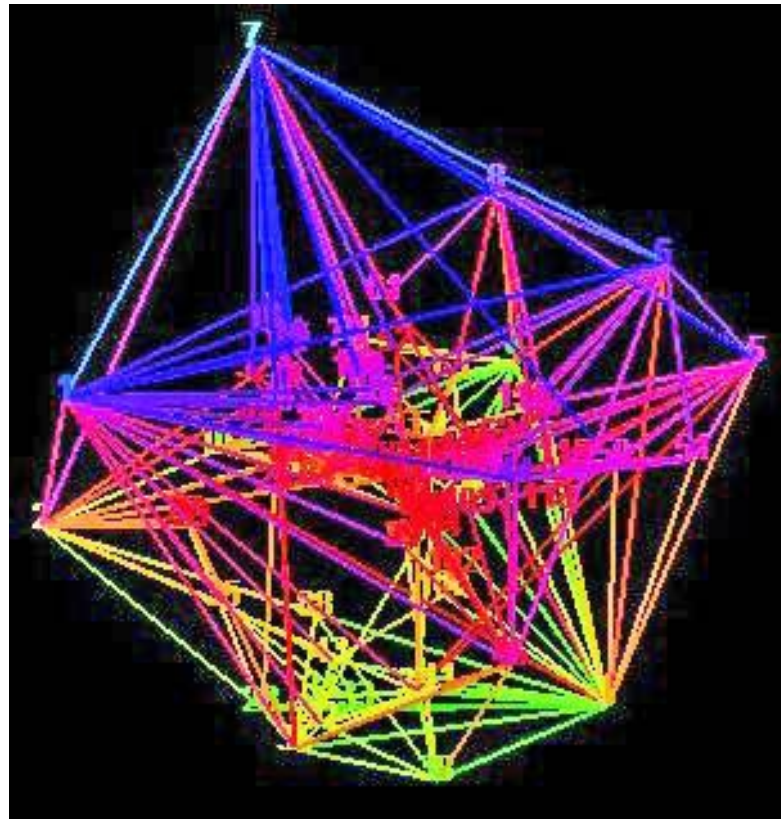
	A 1	A 2	A 3	A 4	A 5	A 6	A 7	A 8	A 9
A 10	A 11	A 12	A 13	A 14	A 15		A 17		A 19
A 20	A 21		A 23	A 24	A 25	A 26	A 27	A 28	A 29
A 30	A 31		A 33				A 37	A 38	A 39
A 40		A 42	A 43	A 44	A 45	A 46		A 48	A 49
		A 52					A 57		A 59
A 60	A 61	A 62	A 63	A 64	A 65	A 66	A 67		
A 70	A 71	A 72	A 73						
	A 81								
		A 92	A 93	A 94	A 95	A 96		A 98	A 99
A 100	A 103	A 104	A 111	A 113	A 114	A 115	A 143		
A 210	A 215	A 226	A 250	A 252	A 253	A 255	A 261		
A 270	A 280	A 281	A 293						
A 352	A 391	A 392	A 395						
A 445	A 480	A 485							
A 516	A 524	A 535	A 540	A 542	A 544	A 553	A 555	A 559	A 560
A 562	A 565	A 571	A 573						
A 602	A 620	A 623	A 643	A 648	A 650	A 656	A 659		
A 661	A 671	A 672							
A 831	A 861	A 864							
A 952	A 980	A 995							

Facts about German Autobahns

- The Autobahns are the nationally coordinated motorway system in Germany. In German, they are called Bundesautobahn (plural Bundesautobahnen, abbreviated BAB), which translates as federal motorway.
German Autobahns have no general speed limit (though about 50% of the total length is subject to local and/or conditional limits), but the recommended speed is 130 km/h maximum, or 80 MPH.
- Today, Germany's Autobahn network has a total length of about 11,980 km (as of January 1, 2003).
- The first German Autobahn was completed in 1932 between Cologne and Bonn.
- The current Autobahn numbering system in use in Germany was introduced in 1974.
- All Autobahns are named by using the capital letter A, which simply stands for "Autobahn" followed by a blank and a number (for example A 8).
- The "main Autobahns" going all across Germany have a single digit number. Shorter Autobahns that are of regional importance (e.g. connecting two major cities or regions within Germany) have a double digit number (e.g. A 24, connecting Berlin and Hamburg). There are also very short Autobahns of just local importance that usually have three numbers, the first one of which is similar to the system above, depending on the region.
- East-west routes are always even-numbered, north-south routes are always odd-numbered.



The 3D dual graph of the network of German Autobahns



What are we interested in?

- Connectivity of the graph
- Components that are highly connected
- Stability (how fast can the network be splitted?)
- Do small worlds exist?
- Spectral properties of the transition operator
- Power laws



Important properties of the graph G

number of vertices in G	104
number of edges in G	468
diameter of G	7
maximal vertexdegree in G	62
minimal vertexdegree in G	2

Symmetric transition operator

- First introduce a random walk with the **transition operator**

$$\tilde{T}_{ij} := \frac{\beta_i}{k_i} A_{i,j} + (1 - \beta_i) \delta_{i,j}$$

which is not symmetric (A is the adjacency matrix of the graph G).

- Consider the **symmetric counterpart T** of the transition operator \tilde{T} :

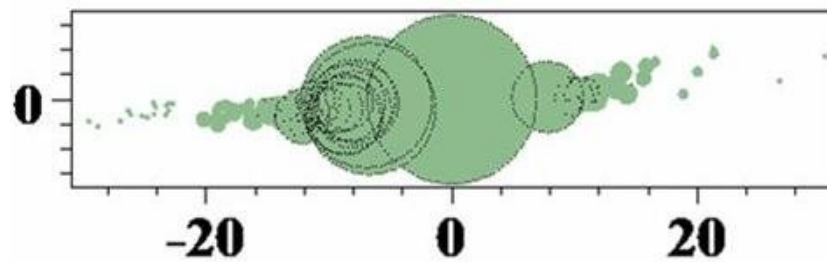
$$T_{ij} := \sqrt{\frac{\beta_i \beta_j}{k_i k_j}} A_{i,j} + (1 - \beta_i) \delta_{i,j}$$

- T has positive spectrum.
- The eigenvectors of T are orthonormal and span the Hilbertspace.



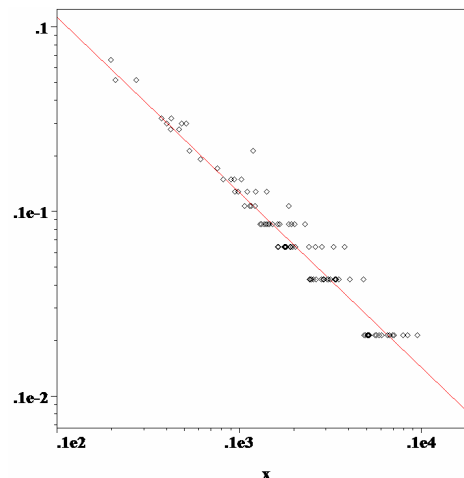
Space syntax representation of German Autobahns

The radii of the disks (vertical axis) show the connectivity of the Autobahns. The expected number of steps from each Autobahn to the Autobahn A 1 is shown on the horizontal axis. By introducing a scalar product it is possible to discuss angles. -> [add powerlaws]



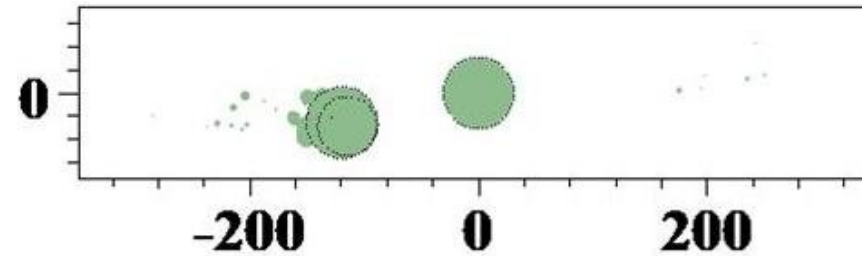
$\beta = 1$ uniformly

Regression with a slope of 0.948

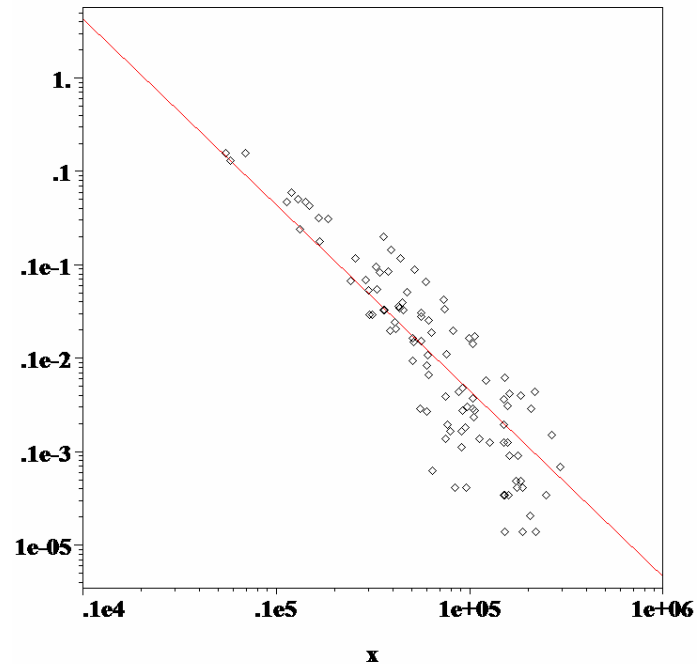




Space syntax representation of German Autobahns



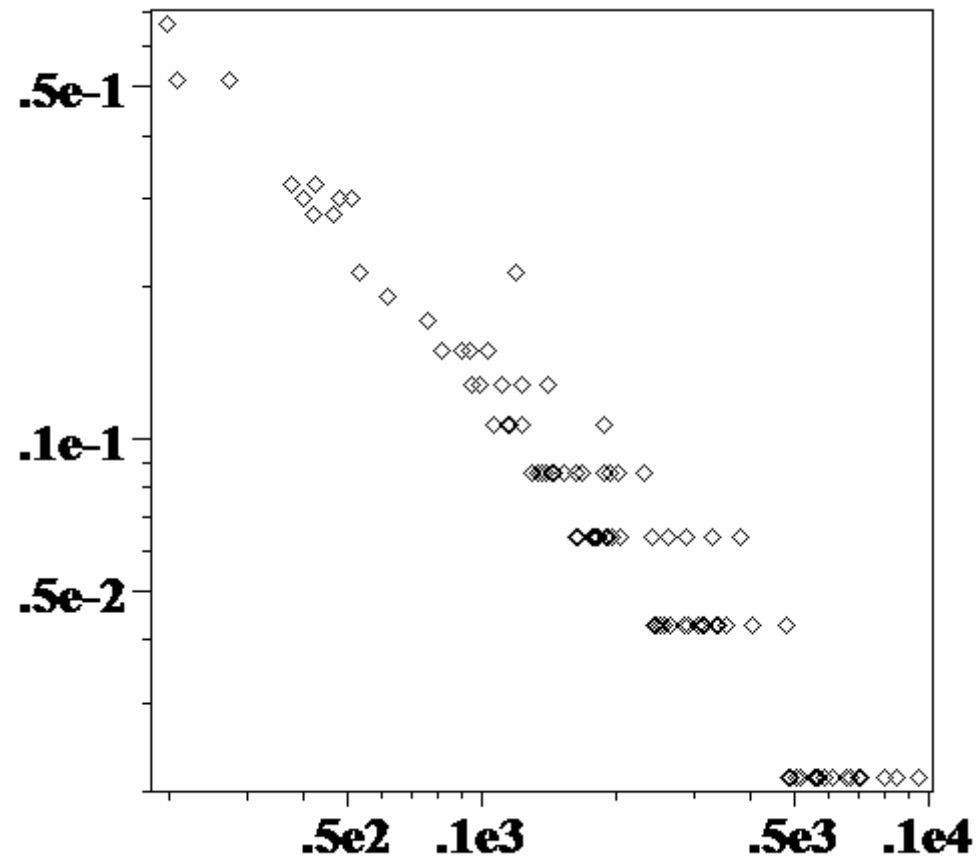
β chosen as the inverted lengths of the Autobahns



Regression with a slope of 1.99

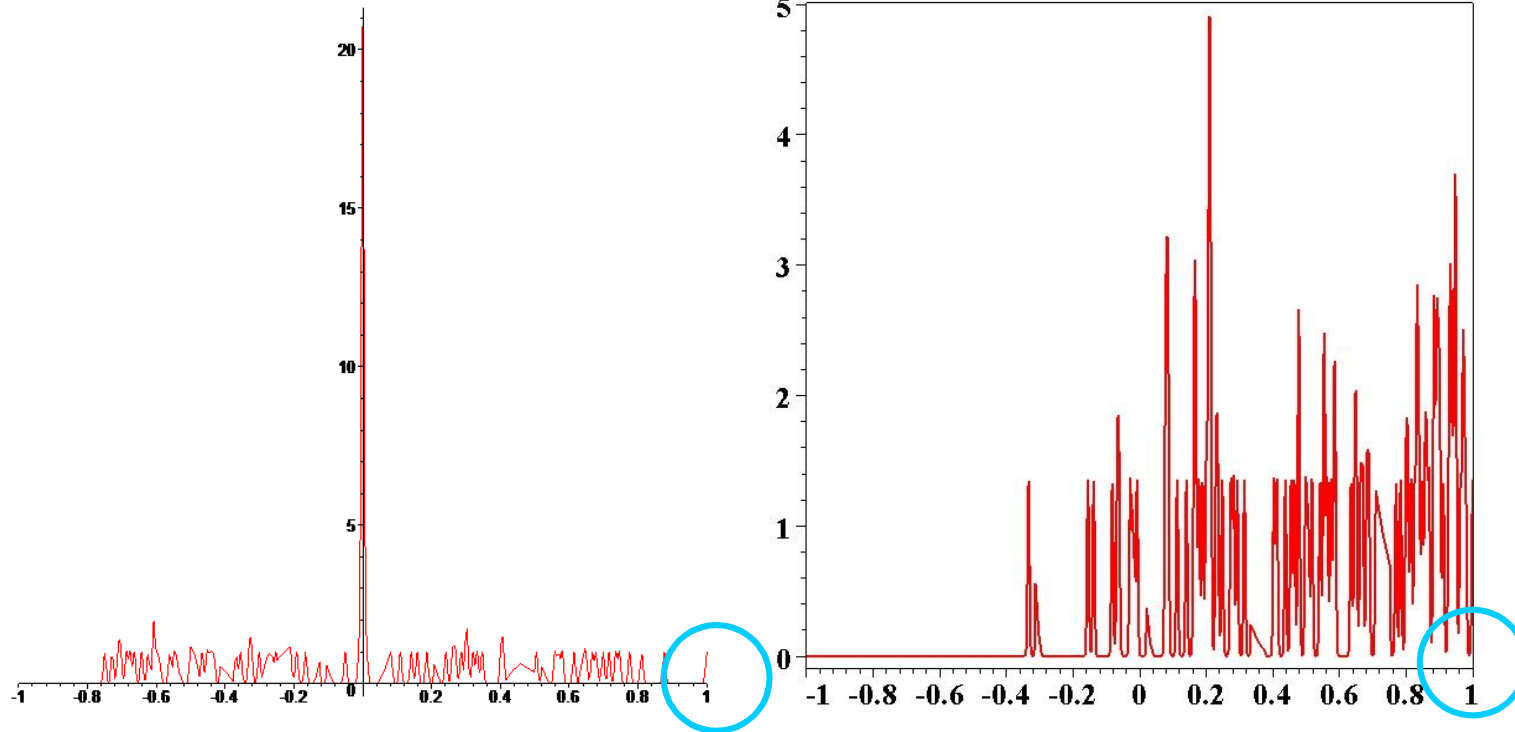


Log-log-plot of vertexdegree k versus number of vertices with vertexdegree k (β is randomized)





Spectral density plots



Case of free traffic flow:
a large spectral gap is observed.
This is an indicator for very good
connectivity of the network.

Case of congestion:
the spectral gap has almost
vanished – an evidence for
splittig the network into
components which are loosely
connected.

Conclusions

- Despite the German Autobahns are highly connected, the the diameter of G is quite big.

$$\text{diam}(G) = 7, \quad \text{diam}(G) \simeq \sqrt{N}$$

This indicates that the entire network does not constitute a small world.

- β does not affect the set of automorphisms of G .
The choice of β has great influence on the power exponent.

$$\mathcal{T}_i \sim \tau_i^{\alpha(\beta_1, \dots, \beta_N)}$$

Therefore the network of German Autobahns is presently evolving and has not reached its equilibrium yet. The vanishing spectral gap would be related to the historical fact of the reunion of Westgermany and the former GDR (DDR) in 1990.